NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

MATHEMATICS with CALCULUS

Level 3

90639 Sketch graphs and find equations of conic sections

Credits: Three

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

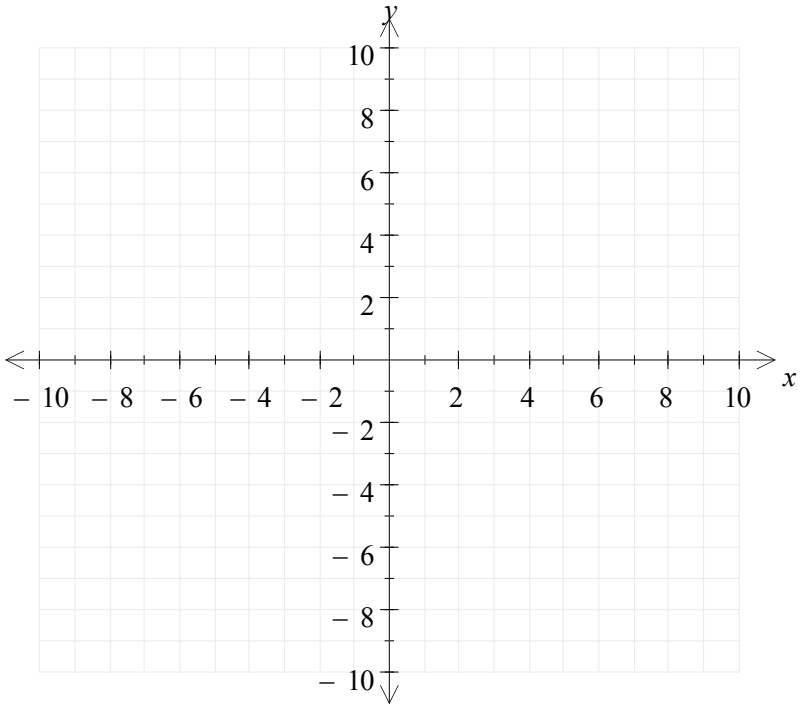
For Assessor's use only **Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Sketch graphs of conic sections. Find equations of conic sections from given information. ☐	Solve problems involving conic sections. ☐	Solve more difficult conic section problems. ☐
Overall Level of Performance ☐		

You are advised to spend 40 minutes answering the questions in this booklet.

QUESTION ONE

Sketch the graph of $x^2 + y^2 + 10y + 16 = 0$.

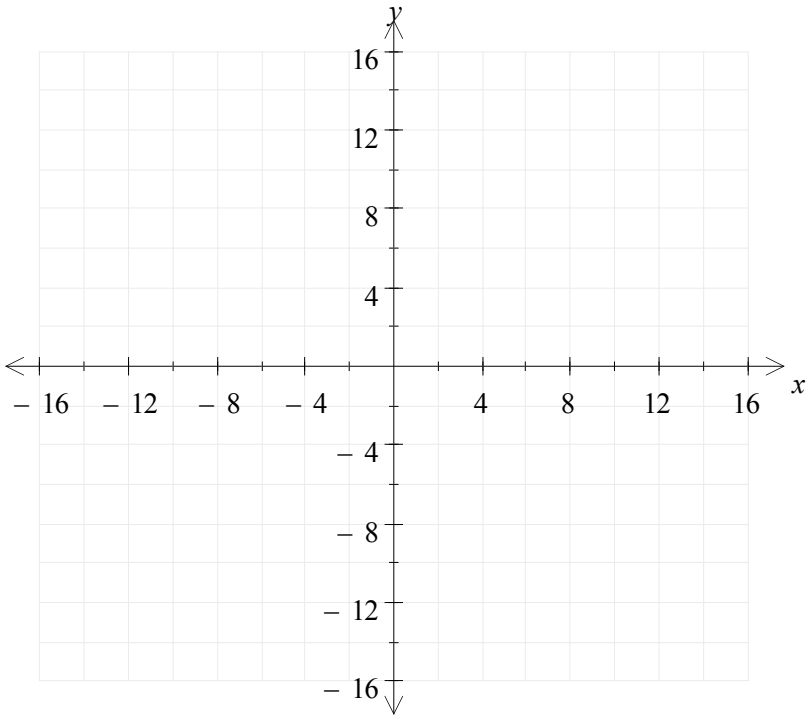
Label the values of any intercepts.



QUESTION TWO

Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

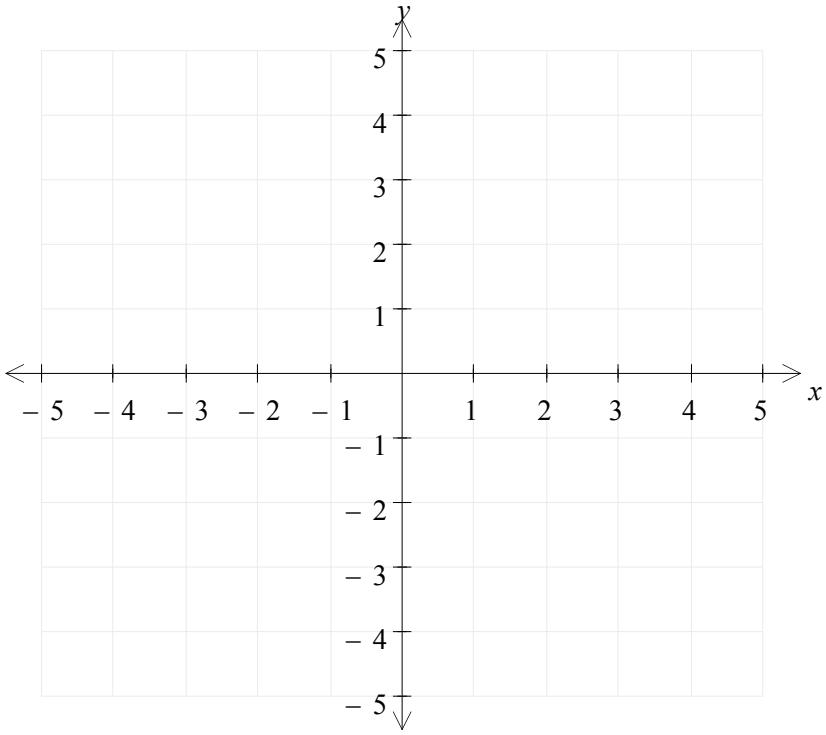
Label the values of any intercepts.



QUESTION THREE

Sketch the graph of $(y-2)^2 = 4x$.

Label the values of any intercepts.

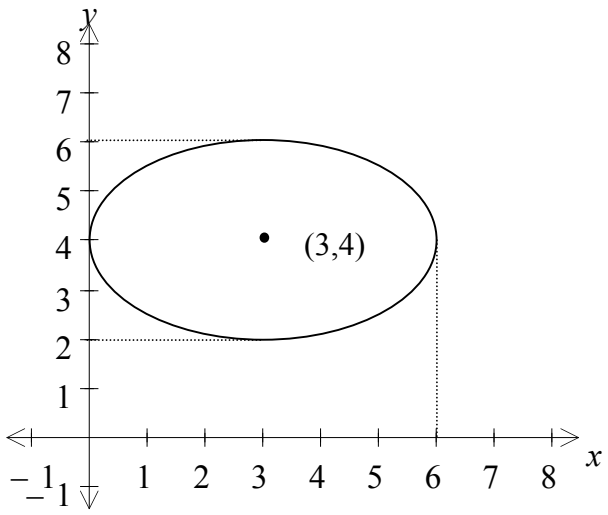


QUESTION FOUR

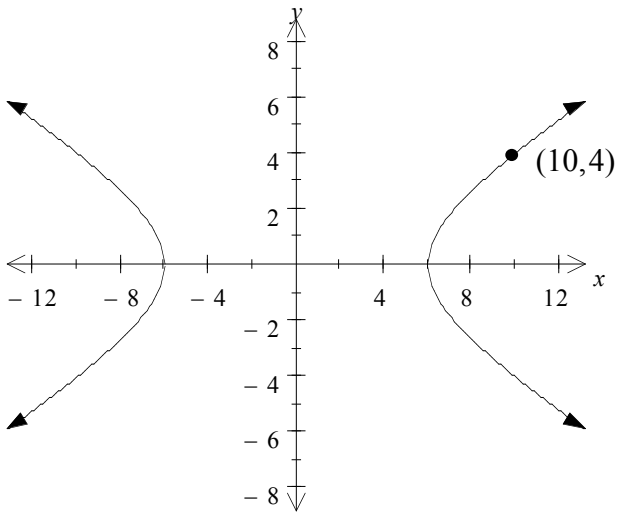
(a) Find the equation of the following conic section:

A circle with centre (4,2) and radius 3.

(b) Find the equation of the following conic section:



(c) Find the equation of the following conic section:



QUESTION FIVE

A curve is defined parametrically as:

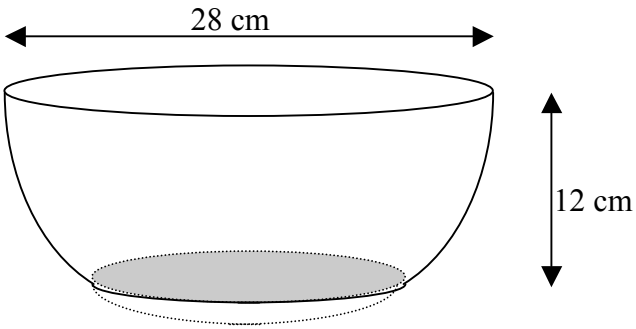
$x = 2\cos 2t$, $y = 2\sin t$.

Find the equation of the tangent to the curve at the point (1,1).

QUESTION SIX

A bowl has the shape of a truncated hemisphere.
The diameter of the top of the bowl is 28 cm.
The depth of the bowl is 12 cm.

Calculate the area of the base of the bowl.



QUESTION SEVEN

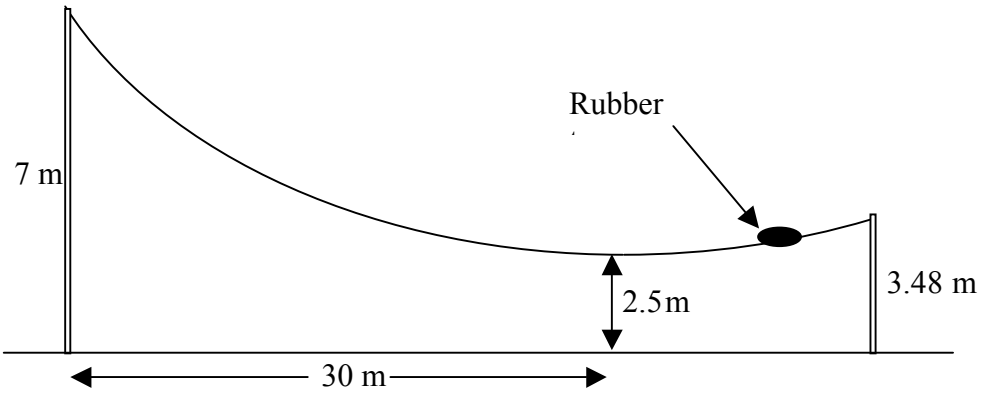
The wire used for a flying fox can be approximately modelled by a parabola.

The wire is fastened to the starting post 7 metres above the ground and to the end post 3.48 metres above the ground.

The lowest point of the wire is 2.5 metres above the ground and 30 metres along from the base of the starting post.

A rubber tyre is attached towards the end of the flying fox wire to stop children from hitting the end post.

How far apart are the two posts?



QUESTION EIGHT

(a) Show that the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

at the point P($a\cos\theta$, $b\sin\theta$) is $ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$.

(b) The normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ at a point Q meets the coordinate axes at A and B respectively.

As Q varies, the locus of the midpoint of AB is another ellipse.

Find the cartesian equation of this ellipse.